Solving Problems Involving Systems of Equations
Note: In solving real-life problems involving a system of equations, follow these four steps.
(i) Clearly identify what the two variables represent (this step is important and counts for marks)
(ii) Set up the two linear equations.
(iii) Solve the system.
(iv) Answer the question posed in the problem.

Sample Problem and Solution:
Two adult and three children’s tickets to an amusement park cost $12.
Three adult and two children’s tickets cost $13. Find the price of each ticket.
Solution:
(i) Let: x = the price of one adult ticket
and y = the price of one children’s ticket.
(ii) Set up the system of equations.
\[2x + 3y = 12\]
\[3x + 2y = 13\]
(iii) Solve the system of equations.
\[4x + 6y = 24\]
\[-9x - 6y = -39\]
\[-5x = -15\]
\[x = 3\]
Substitute x = 3 in the first equation to obtain:
\[2(3) + 3y = 12\]
\[3y = 12 - 6\]
\[y = 2\]
(iv) An adult ticket costs $3 and a children’s ticket costs $2.

Solve the following problems using the four steps.
1. Paul bought 2 pens and 4 duotangs for $5. Tanya bought 3 pens and 5 duotangs for $7. Linda bought 1 pen and 1 duotang. All shopped at the same store for similar pens and duotangs. Find the system of linear equations and determine the amount paid by Linda.

2. One printing company charges $500 plus $1.50 for printing each copy of the yearbook. Another company charges $800 plus $1 for printing each copy. The student council calculated that no matter which printing company they chose, the printing charges would be the same. How many copies of the yearbook need to be printed?
3. Last week Doreen earned $140 from her part-time job at a restaurant. She was paid in twenty-dollar and ten-dollar bills. The number of twenty-dollar bills was 4 more than the number of ten-dollar bills. Find the number of ten-dollar bills.

\[x = \# \text{ of } 20\text{-dollar bills} \]
\[y = \# \text{ of } 10\text{-dollar bills} \]

Value
\[10x + 20y = 140 \]
\[-x + y = 4 \]
\[0x + 20y = 180 \]
\[10x + 20(x + 4) = 140 \]

There are 2 ten-dollar bills.

4. Peter and John took part in a cross-country race. The sum of twice the distance run by Peter and three times the distance run by John was 22 km. The difference of three times the distance run by Peter and twice the distance run by John was 7 km. Find the distance each boy ran.

Peter ran \( \frac{1}{3}\) km, John ran \( \frac{4}{3}\) km.

5. Shingles Roofing company charges $900 for materials and $80 per hour for a job. Hargreaves Roofers charges $1200 plus $65 per hour for a job. If a job costs the same for both companies, how long did the job take?

It took 20 hours.

6. A total of 620 people attended the Centre City High School athletics events. The admission fee was $2.50 for an adult and $1.50 for a child. A total of $1150 was collected from ticket sales. How many children attended the events?

450 children attended.
7. Kelly worked 50 hours last week. Her regular wage is $6 per hour. She is paid $9 an hour for overtime. Last week she earned $360. How many hours of overtime did she work last week?

\[ \begin{align*}
\text{Regular hours:} & \quad \begin{cases} x + y = 50 \\ 6x + 9y = 360 \end{cases} \\
\text{Overtime hours:} & \quad \begin{cases} -6x - 6y = -300 \\ 2y = 60 \end{cases}
\end{align*} \]

Solving, we get:

\[ \begin{align*}
x + 20 &= 50 \\
x &= 30
\end{align*} \]

\[ \begin{align*}
y &= 20
\end{align*} \]

She worked 20 hours of overtime.

8. A department store is having a suit sale in which jackets and pants are sold at different prices. Sally bought 3 jackets and 2 pairs of pants for $210. Her friend Carol bought 2 jackets and 3 pairs of pants for $190. How much does each cost?

\[ \begin{align*}
x &= \text{price for a jacket} \\
y &= \text{price for a pants}
\end{align*} \]

\[ \begin{align*}
3x + 2y &= 210 \\
2x + 3y &= 190
\end{align*} \]

Solving the system:

\[ \begin{align*}
3x + 2y &= 210 \\
2x + 3y &= 190
\end{align*} \]

\[ \begin{align*}
3x + 2y &= 210 \\
2x + 3y &= 190
\end{align*} \]

\[ \begin{align*}
6x + 4y &= 420 \\
-4x - 6y &= -340
\end{align*} \]

\[ \begin{align*}
2y &= 80 \\
y &= 40
\end{align*} \]

A jacket costs $50 and the pants cost $30.

9. At a fruit market, Jill bought 7 boxes of apples and 5 boxes of oranges for $41, whereas her friend Harry bought 7 boxes of apples and 8 boxes of oranges for $53. What is the cost of one box of apples and one box of oranges?

\[ \begin{align*}
x &= \text{price for a box of apples} \\
y &= \text{price for a box of oranges}
\end{align*} \]

\[ \begin{align*}
7x + 5y &= 41 \\
7x + 8y &= 53
\end{align*} \]

\[ \begin{align*}
7x + 5(4) &= 41 \\
7x + 20 &= 41
\end{align*} \]

\[ \begin{align*}
x &= 3
\end{align*} \]

\[ \begin{align*}
x &= 21
\end{align*} \]

A box of apples costs $3 and a box of oranges costs $4.

10. A school is organizing a lottery to raise money to buy a laser printer. The treasurer of the student council determines that if each ticket is sold for $1, they will still need $400. But if each ticket is sold for $1.50, they will have $60 left over after having bought the printer.

How many tickets must be sold to buy the printer? What is the price of the printer?

\[ \begin{align*}
x &= \text{number of tickets sold} \\
y &= \text{price of the printer}
\end{align*} \]

\[ \begin{align*}
x + y &= 920 \\
y &= 1.5x - 60
\end{align*} \]

\[ \begin{align*}
x + y &= 920 \\
y &= 1.5x - 60
\end{align*} \]

\[ \begin{align*}
y &= x + 400 \\
y &= 920 + 400
\end{align*} \]

\[ \begin{align*}
1320 &= x
\end{align*} \]

They must sell 920 tickets. The printer costs $1320.
11. Sofia has $11 more than Vanna. Sofia spent $9 and Vanna spent $12 on a dinner. If they still have $141 between them, how much did each have in the beginning?

\[ \begin{align*}
\text{Sofia} & = y + 11 \\
\text{Vanna} & = y - 12 \\
\text{Sofia had } & = 86.50 \quad \text{and} \\
\text{Vanna had } & = 75.50
\end{align*} \]

\[ (x - 9) + (y - 12) = 141 \\
\text{x + y - 21 = 141} \\
x = 162 - y \\
\text{x = 75.50 + 11} \\
\text{e = 86.50} \]

12. The perimeter of a rectangular field is 22 m. Its length is 1 m less than twice its width.

What is the length of the field? Make a diagram.

\[ \begin{align*}
\text{w} & = \text{width (m)} \\
\text{l} & = \text{length (m)} \\
2l + 2w & = 22 \\
3l & = 21 \\
l & = 7
\end{align*} \]

The length is 7 m

13. The cost of transporting a car on a ferry is a set price for the driver plus an additional charge for each passenger. The driver of a Volkswagen with two passengers paid $24. The driver of a BMW with five passengers paid $33. How much will it cost the driver of a Ford with three passengers?

\[ \begin{align*}
\text{J} & = \text{cost per car} \\
\text{y} & = \text{cost per passenger} \\
\text{x} + 2y & = 24 \\
\text{x} + 5y & = 33 \\
\text{x} + 2(3) & = 2y \\
\text{x} & = 24 - 6 \\
\text{x} & = 18 \\
\text{Ford} & = 18 + 3(3) = \$27
\end{align*} \]

14. Judy has $35 in her bank account and decides to save $8 each week. Mary has $53 in her account but wants to save $5 per week.

Given: \( x = \) the number of weeks

\[ \begin{align*}
\text{J}(x) & = \text{Judy's total savings in } x \text{ weeks} \\
\text{M}(x) & = \text{Mary's total savings in } x \text{ weeks}
\end{align*} \]

Find the system of equations that determines the savings of each.

Determine when they will have the same amount of savings by making \( \text{J}(x) = \text{M}(x) \).

\[ \begin{align*}
\text{They will have the} & \text{ same after } 6 \text{ weeks}
\end{align*} \]