A function is a relation between an input value $x$, and the related output value, $y$. There is only one $y$-value possible for any $x$-value.

Consider the linear equation $y = \frac{1}{2}x - 2$ shown in the graph at the right.

The independent variable is $x$, and the dependent variable is $y$. We say $y$ is a function of $x$.

Function notation for the line is $f(x) = \frac{1}{2}x - 2$.

This is how it works… remember that a line is a set of points. $f$ is the “name” of the function.

Consider the point on the line $(8, 2)$. This means that at $x = 8$, the related $y$-value is 2.

In function notation, we say that the function value at $x = 8$ is 2, and write it as $f(8) = 2$.

**Function Notation on a Graph**

Graphically, $f(8)$ means, the height of the function at $x = 8$. Go to 8 on the $x$-axis, and locate the point on the line directly above (or below) it. The $y$-value of the point is 2.

Try these:  
Find $f(6)$  
Find $f(2)$  
Find $f(-4)$  
Find $f(11)$

**Function Notation with Algebra**

Algebraically, $f(8)$ means plug $x = 8$ into the rule, and calculate the $y$-value.

Using $f(x) = \frac{1}{2}x - 2$, then $f(8) = \frac{1}{2}(8) - 2$ 
$= 4 - 2$ 
$= 2$ so, we can say $f(8) = 2$.

Try these:  
Calculate $f(6) = \frac{1}{2}(6) - 2$  
Calculate $f(-2) = \frac{1}{2}(-2) - 2$  
Calculate $f(11) = \frac{1}{2}(11) - 2$
Look at the two functions graphed at the right.

Find the following:

\[ f(1) = 4 \quad f(-1) = -4 \quad f(0) = 0 \]

\[ g(5) = 7 \quad g(-1) = 9 \quad g(8) = 6 \]

For what value is \( f(x) = g(x) \)?

\[ x = 2 \]

For what value is \( f(x) = 12 \)?

\[ x = 3 \]

For what value is \( g(x) = 10 \)?

\[ x = 4 \]

For the “nice” points, shown on the graph, answering these questions should be easy. But what if you were asked for \( f(10) \) or \( g(3) \)? We need to use the rule of the functions, so that we can calculate the \( y \)-value.

Write the rules for the functions:

\[ f(x) = \frac{4}{3}x + 0 \quad g(x) = -\frac{1}{2}x + 8 \]

Now calculate the following:

\[ f(10) = 4(10) = 40 \]

\[ g(3) = -\frac{1}{2}(3) + 8 \]

\[ f(-3.5) = 4(-3.5) = -14 \]

\[ g(-25) = -\frac{1}{2}(-25) + 8 \]

For what value is \( f(x) = 20 \)?

\[ \frac{4}{3}x = 20 \]

\[ x = 15 \]

For what value is \( g(x) = 20 \)?

\[ \frac{1}{2}x = 11 \]

\[ x = -22 \]

These are the backwards kind of questions
Here is a list of the types of functions we need to study (in order of complication).

\[ k(x) = 4 \]  **Constant function** – no matter what is input the output is always the same

The graph is a horizontal line

\[ j(x) = x \]  **Identity function** – the output value is equal to the input value

The graph is a line passing through the origin with a 45° angle

\[ f(x) = \frac{3}{2} x - 5 \]  **Linear function** – multiply by a factor and then add a constant

The graph is a line with slope \( \frac{3}{2} \) and a y-intercept of -5

\[ g(x) = 3x^2 \]  **Quadratic function** – square the input and then multiply by the factor

The graph is a parabola (2\(^{nd}\) degree equation, not a line)

We will also study variations of this function

\[ h(x) = 2[x] \]  **Greatest Integer function** – round the number down, then multiply by the factor

The graph is a staircase with a slope of 2.

We will also study variations of this function

\[ e(x) = 3^x \]  **Exponential function** – multiply the base by itself \( x \) times

The graph is an increasingly steep curve.

Calculate the following using the functions given above:

1. \( e(4) \)  
2. \( f(3) \)  
3. \( g(5) \)  
4. \( h(1.7) \)  
5. \( j(1.5) \)  
6. \( k(1.5) \)
What is a function? You tell something to perform a particular function (operation) and it does it. Vacuum the living room. What is the function? What is the domain? What is the range?

Square this number. What is the function? What is the domain? What is the range?

I’ll pay a taxi driver to drive me to where I want to go. What is the function? What is the domain? What is the range?

A function is an operation that is performed on the elements of the domain. The range is the set of possible outcomes/results.

Usually the Domain is the set of all Real numbers. The Function is a combination of operations (add, subtract, multiply, divide, exponent). The Range is the set of outcomes/answers from the operations. Here are a few examples: